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On the relationship between impulse response analysis, innovation accounting and Granger causality

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Abstract

We derive a general characterization of Granger non-causality between two vectors, X_1 and X_2, in terms of impulse responses, when a third vector of auxiliary variables, X_3, is also used to forecast. It is observed that the usual characterizations of non-causality in terms of impulse responses or variance decompositions can be misleading in such situations. A numerical illustration is also provided.

1. Introduction

Since the classical contributions of Sims (1980a, b, 1982), vector autoregressive models (VAR) have been widely used to study the dynamic structure of economic time series. Especially important properties that are analyzed from such models include causality in the sense of Wiener (1956) and Granger (1969), impulse responses (coefficients of moving average representations) and innovation accounting (variance decompositions). These properties are linked. In particular, it is often argued that non-zero impulse responses (after an appropriate orthogonalization, if required) indicate the presence of Granger causality, while variance decompositions yield natural measures of Granger causal priority. For example, while discussing VAR systems with three and six variables, Sims (1982, pp. 131–132) states: 'A natural measure of the degree to which Granger causal priority holds is the percentage of forecast error variance accounted for by a variable's own future disturbances in a multivariate linear autoregressive model . . . . A variable that is optimally forecast from its own lagged values will have all its forecast error variance accounted for by its own disturbances'; see also Sims (1980b, pp. 251–252) for a similar statement. Several other authors who have analyzed multivariate VAR models with more than two variables have also used the same relationship between variance decompositions and Granger causality; see, for example,

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McMillin (1988, pp. 325–326), Faroque and Veloce (1990, p. 281), Kyereme (1991, pp. 1807–1808), Stam et al. (1991, pp. 214 and 225), and Tegene (1991, p. 1374).

The bivariate stationary systems, the equivalence between Granger non-causality from a variable X_1 to a variable X_2 ($X_1 \nrightarrow X_2$) and the nullity of the coefficients of the innovations of X_1 in the bivariate moving average (MA) representation of X_2 has been established by Sims (1972); see also Pierce and Haugh (1977, Theorem 4.2). It is then straightforward to see that the proportion of the variance of X_2 accounted for by the innovations of X_1 must be zero. Furthermore, it is easy to obtain a similar result when X_1 and X_2 are vectors; see Caines and Chan (1975) and Lütkepohl (1991, section 2.3). Consequently, whenever a stationary process has an autoregressive (AR) representation, there is a simple duality between characterizations of non-causality based on AR coefficients and those based on MA coefficients. On the other hand, as the quotes and references given above clearly show, it is not widely recognized that the simple linear MA characterization of non-causality holding in bivariate systems does not extend to multivariate systems which include variables other than X_1 and X_2. In particular, no general MA characterization of non-causality for such systems appears to be available.

The main purpose of this paper is to give a general necessary and sufficient condition for non-causality in terms of MA coefficients between two vectors (or variables) inside a larger system that may include other variables as well. The setup considered is the class of second-order stationary and invertible (strictly indeterministic) stochastic processes, i.e. stationary indeterministic processes possessing an autoregressive representation (possibly of infinite order). The condition given involves non-linear restrictions on the MA coefficients and clearly shows that the duality between AR and MA characterizations of non-causality, which occurs in bivariate systems, does not generally hold in multivariate systems, and why it is so. In particular, even if X_1 does not cause X_2 in the sense of Granger, the innovations of X_1 may account for a sizeable proportion of the variance of X_2. Conversely, even if the latter proportion is zero, it is quite possible that X_1 causes X_2. A simple numerical example illustrating such situations is given below. These complications occur irrespective of whether the innovations of the process are contemporaneously correlated (or not) and so are quite distinct from the familiar problems associated with the choice of an orthogonalization for the innovations of the model. We also deduce from our characterization a number of simpler (linear) sufficient conditions based on MA coefficients (or impulse responses) as well as a necessary condition which only depends on the first impulse coefficient.

In section 2 we give the required definitions and describe the setup considered. In section 3 we derive the conditions for non-causality proposed. A numerical illustration is presented in section 4. We conclude in section 5 by discussing some practical implications of our results.

2. Framework

Let $\{X(t): t \in \mathbb{Z}\}$ be an $m \times 1$ discrete-time vector stochastic process which possesses a moving average representation of the form

$$X(t) = \sum_{k=0}^{\infty} \psi_k a(t-k) = \psi(B)a(t), \quad t \in \mathbb{Z}, \tag{1}$$

where ψ_k, $k \geq 0$, are $m \times m$ matrices such that $\sum_{k=0}^{\infty} \|\psi_k\|^2$ is finite and $\psi_0 = I_m$, $\|\psi_k\|^2 = \mathrm{tr}(\psi_k \psi_k')$, $\psi(B) = \sum_{k=0}^{\infty} \psi_k B^k$ is a matrix of formal series in the lag operator B, $\{a(t): t \in \mathbb{Z}\}$ is a white noise process with non-singular covariance matrix, i.e. the vectors $a(t)$ are mutually uncorrelated such that $E[a(t)] = 0$ and $\det(V[a(t)]) \neq 0$; we also suppose that $a(t)$ is the innovation process of $X(t)$.

Furthermore, let us partition $X(t)$ and $a(t)$ into three subvectors: $X(t) = (X_1(t)', X_2(t)', X_3(t)')'$ and $a(t) = (a_1(t)', a_2(t)', a_3(t)')'$, where $X_i(t)$ and $a_i(t)$ have dimension $m_i \times 1$ with $m_1 \geq 1$, $m_2 \geq 1$, $m_3 \geq 0$ and $m_1 + m_2 + m_3 = m$. When $m_3 = 0$, the partition involves only two subvectors.

We say that the process $X(t)$ is *invertible* if it can be written in autoregressive form:

$$\Pi(B)X(t) = a(t), \quad t \in \mathbb{Z}, \tag{2}$$

where the matrix $\Pi(B)$ is defined by $\Pi(B) = \psi(B)^{-1} = I_m - \sum_{k=1}^{\infty} \Pi_k B^k$, $\sum_{k=1}^{\infty} \|\Pi_k\| < \infty$, and we assume that the equation $\det[\psi(z)] = 0$ has all its roots outside the unit circle ($z \in \mathbb{C}$). Furthermore, in the context of model (1), we say that X_1 *does not cause* X_2 in the sense of Granger (noted $X_1 \nrightarrow X_2$) if the following identity holds with probability 1:

$$P[X_2(t+1)|\bar{X}(t)] = P[X_2(t+1)|\bar{X}_2(t), \bar{X}_3(t)], \quad \text{for all } t, \tag{3}$$

where $\bar{X}(t) = (X(\tau): \tau \leq t)$, $\bar{X}_i(t) = (X_i(\tau): \tau \leq t)$ and $P[X_2(t+1)|\cdot\,]$ is the vector of the best linear predictors (in the mean squre sense) of the components of $X_2(t+1)$ based on the list of variables appearing after the bar '|' (i.e. the linear projection operator). It is easy to see that (3) is equivalent to either one of the two following conditions:

$$V[\epsilon(X_2(t+1)|\bar{X}(t))] = V[\epsilon(X_2(t+1)|\bar{X}_2(t), \bar{X}_3(t))], \quad \text{for all } t, \tag{4}$$

$$V[\epsilon(x_{2i}(t+1)|\bar{X}(t))] = V[\epsilon(x_{2i}(t+1)|\bar{X}_2(t), \bar{X}_3(t))], \quad i = 1, \ldots, m_2, \quad \text{for all } t, \tag{5}$$

where $\epsilon(X_2(t+1)|\cdot) = X_2(t) - P[X_2(t)|\cdot\,]$, $X_2(t) = [x_{2i}(t): i = 1, \ldots, m_2]'$ and $\epsilon(X_2(t+1)|\cdot) = [\epsilon(x_{2i}(t+1)|\cdot): i = 1, \ldots, m_2]'$; see Boudjellaba et al. (1992).

Let us now partition ψ_k, $\psi(B)$ and $\Pi(B)$ conformably with the partitions of $a(t)$ and $X(t)$:

$$\psi(B) = [\psi_{ij}(B)]_{i,j=1,2,3,}, \qquad \Pi(B) = [\Pi_{ij}\ B)]_{i,j=1,2,3}, \tag{6}$$

with $\psi_{ij}(B) = \sum_{k=0}^{\infty} \psi_{ijk} B^k$, where ψ_{ijk}, $\psi_{ij}(B)$ and $\Pi_{ij}(B)$ have dimensions $m_1 \times m_j$ $(i, j = 1, 2, 3)$, $\psi_{ij0} = I_{m_i}$ if $i = j$, and $\psi_{ij0} = 0$ if $i \neq j$. When the process $X(t)$ is invertible and no auxiliary variables are present ($m_3 = 0$), it is easy to see that $\Pi_{21}(z) = 0$ if and only if $\psi_{21}(z) = 0$, where $|z| \leq 1$. For any positive real constant $\delta (0 < \delta \leq 1)$, the condition $\Pi_{21}(z) = 0$ for $|z| \leq \delta$ is equivalent to stating that each coefficient in the power series $\sum_{k=1}^{\infty} \Pi_{21k} z^k$ is zero, i.e. $\Pi_{21k} = 0$ for $k = 1, 2, \ldots,$ and similarly for other conditions of the same form, such as $\psi_{21}(z) = 0$ for $|z| \leq 1$. Furthermore, the condition $\Pi_{21}(z) = 0$ for $|z| \leq 1$ is necessary and sufficient for $X_1 \nrightarrow X_2$ [see Boudjellaba et al. (1992, Proposition 1)], so that

$$X_1 \nrightarrow X_2 \Leftrightarrow \psi_{21}(z) \text{ for } |z| \leq 1 \Leftrightarrow \psi_{21k} = 0, \quad k = 1, 2, \ldots, . \tag{7}$$

Characterization (7) of non-causality is well known [see Lütkepohl (1991, section 2.3)] and allows a direct causality interpretation of the impulse response coefficients ψ_{21k}. However, such a simple interpretation no longer holds when auxiliary variables are present ($m_3 \geq 1$).

3. Moving average characterizations of non-causality

In the following proposition we give a general characterization of Granger non-causality from X_1 to X_2 ($X_1 \nrightarrow X_2$) in terms of the coefficients of the MA representation (1), when $m_3 \geq 1$.

Proposition 1. Let $X(t)$ be a second-order stationary process which satisfies (1), *let $m_3 \geq 1$, and let δ be any real constant such that $0 < \delta \leq 1$ and $\det[\psi_{33}(z)] \neq 0$ for $|z| < \delta$. Then $X_1 \nrightarrow X_2$ if and only if*

$$\psi_{21}(z) - \psi_{23}(z)\psi_{33}(z)^{-1}\psi_{31}(z) = 0, \quad \text{for } |z| < \delta . \tag{8}$$

Proof. By the invertibility condition, the process $X(t)$ satisfies Eq. (2) where $\Pi(z) = \psi(z)^{-1}$ for $|z| \leq 1$. Furthermore, by Proposition 1 of Boudjellaba et al. (1992) we have

$$X_1 \nrightarrow X_2 \Leftrightarrow \Pi_{21}(z) = 0, \quad \text{for } |z| \leq 1 .$$

Let $A_{22}(z) = [\psi_{ij}(z)]_{i,j=2,3}$ and $\psi_{22\cdot 3}(z) = \psi_{22}(z) - \psi_{23}(z)\psi_{33}(z)^{-1}\psi_{32}(z)$. Since $\psi_{ii}(0) = I_{m_i}$, $i = 1, 2, 3$, and $\psi_{ij}(0) = 0$ for $i \neq j$, the matrices $\psi_{ii}(z)$, $i = 1, 2, 3$, $A_{22}(z)$ and $\psi_{22\cdot 3}(z)$ are all invertible for $|z|$ sufficiently small, say for $|z| < \delta_0$, where $0 < \delta_0 < 1$. Let $A_{22}(z)^{-1} = [\tilde{\psi}^{ij}(z)]_{i,j=2,3}$, where $\tilde{\psi}^{ij}(z)$ is an $m_i \times m_j$ matrix ($|z| < \delta_0$). By standard formulas for the inversion of partitioned matrices [see Graybill (1983, section 8.2)], we see that, for $|z| < \delta_0$ (dropping the symbol z to simplify the notation),

$$\begin{bmatrix} \Pi_{21} \\ \Pi_{31} \end{bmatrix} = -A_{22}^{-1} \begin{bmatrix} \psi_{21} \\ \psi_{31} \end{bmatrix} \tilde{\psi}_{11}^{-1} = -\begin{bmatrix} \tilde{\psi}^{22} & \tilde{\psi}^{23} \\ \tilde{\psi}^{32} & \tilde{\psi}^{33} \end{bmatrix} \begin{bmatrix} \psi_{21} \\ \psi_{31} \end{bmatrix} \tilde{\psi}_{11}^{-1} ,$$

where $\tilde{\psi}_{11}^{-1} = \psi_{11} - \tilde{\psi}_{12} A_{22}^{-1} \tilde{\psi}_{21}$ with $\tilde{\psi}_{12} = [\psi_{12}, \psi_{13}]$ and $\tilde{\psi}_{21} = [\psi_{21}', \psi_{31}']'$, $\tilde{\psi}^{22} = \psi_{22\cdot 3}^{-1}$ and $\tilde{\psi}^{23} = -\psi_{22\cdot 3}^{-1}\psi_{23}\psi_{33}^{-1}$; hence

$$\Pi_{21} = -\psi_{22\cdot 3}^{-1}[\psi_{21} - \psi_{23}\psi_{33}^{-1}\psi_{31}]\tilde{\psi}_{11}^{-1} .$$

Thus, $\Pi_{21} = 0$ for $|z| < \delta_0$ if and only if $\psi_{21} - \psi_{23}\psi_{33}^{-1}\psi_{31} = 0$ for $|z| < \delta_0$. Furthermore, by the unicity of the coefficients of convergent power series, identity (8) holds for $|z| < \delta_0$ if and only if it also holds for $|z| < \delta$, where δ is any constant such that $0 < \delta \leq 1$ and $\det[\psi_{33}(z)] \neq 0$. *Q.E.D.*

In Proposition 1, it is clear (by continuity) that the constant δ always exists because $\psi_{33}(0) = I_{m_3} \neq 0$. The relevant restrictions on the matrices ψ_k, $k \geq 1$, are obtained by setting equal to zero the coefficients of the formal series expansion of $\psi_{21}(z) - \psi_{23}(z)\psi_{33}(z)^{-1}\psi_{31}(z)$. Condition (8) implies that $\psi_{21}(z) = 0$ is neither necessary nor sufficient for $X_1 \nrightarrow X_2$. Causality from X_1 to X_2 also depends on the impulse responses of the innovations of X_1 on $X_3(\psi_{31})$, those of X_3 on X_2 (ψ_{23}), and the own impulse responses of X_3 (ψ_{33}): causality from X_1 to X_2 may be due to (or cancelled by) the joint effect of the innovations of X_1 on X_3 and the innovations of X_3 on X_2. However, it is easy to derive from (8) relatively simple sufficient conditions for $X \nrightarrow X_2$. The following corollary provides such conditions.

Corollary 1. Under the assumptions of Proposition 1, each one of the three following conditions is sufficient for $X_1 \nrightarrow X_2$:

(a) $$\psi_{21}(z) = 0 \text{ and } \psi_{23}(z)\psi_{33}(z)^{-1}\psi_{31}(z) = 0, \quad \text{for } |z| < \delta ; \tag{9}$$

(b) $$\psi_{21}(z) = 0 \text{ and } \psi_{23}(z) = 0, \quad \text{for } |z| < \delta ; \tag{10}$$

(c) $$\psi_{21}(z) = 0 \text{ and } \psi_{31}(z) = 0, \quad \text{for } |z| < \delta . \tag{11}$$

Condition (10) can be interpreted as the case where the innovations of both X_t and X_3 have no effect on X_2, while condition(11) is the case where the innovations of X_1 have no effect on both X_2 and X_3. In these cases, the matrix $\psi(B)$ can be put in a block-triangular form (after permuting either X_1 and X_2, or X_1 and X_3). Conversely, if we observe that

$$\psi_{23}(z)\psi_{33}(z)^{-1}\psi_{31}(z) = \Big[\sum_{k=1}^{\infty} \psi_{23k}z^k\Big]\Big[\sum_{k=0}^{\infty} \psi_{33k}z^k\Big]^{-1}\Big[\sum_{k=1}^{\infty} \psi_{23k}z^k\Big] = \sum_{k=2}^{\infty} c_k z^k ,$$

we can see from (8) that ψ_{211}, the coefficient of $a_1(t-1)$ in the MA representation of $X_2(t)$, must be zero to have $X_1 \nrightarrow X_2$: $\psi_{211} = 0$ is a necessary condition for $X_1 \nrightarrow X_2$ (but $\psi_{21k} = 0$, for $k \geq 2$, is not).

Corollary 2. Under the assumptions of Proposition 1, the condition $\psi_{211} = 0$ is necessary for $X_1 \nrightarrow X_2$.

Since the condition $\psi_{21}(z) = 0$ is neither necessary nor sufficient for $X_1 \nrightarrow X_2$, it remains of interest to know which restrictions on $\Pi(z)$ are equivalent to $\psi_{21}(z) = 0$. These can be obtained easily if we replace $\psi(B)$ by $\Pi(B) = \psi(B)^{-1}$ in model (1), yielding the following proposition.

Proposition 2. Let the assumptions of Proposition 1 *hold, and let δ_0 be any real constant such that $0 < \delta_0 \leq 1$ and $\det[\Pi_{33}(z)] \neq 0$ for $|z| < \delta_0$. Then $\psi_{21}(z) = 0$ for $|z| < \delta_0$ if and only if*

$$\Pi_{21}(z) - \Pi_{23}(z)\Pi_{33}(Z)^{-1}\Pi_{31}(z) = 0, \quad \text{for } |z| < \delta_0 .$$

4. Numerical example

To illustrate that AR and MA coefficients can suggest very different ‘causality’ interpretations, consider the following simple trivariate AR(1) model:

$$X(t) - \Pi_1 X(t-1) = a(t), \quad t \in \mathbb{Z} , \tag{12}$$

where $X(t) = (X_1(t), X_2(t), X_3(t))'$ and $a(t) = (a_1(t), a_2(t), a_3(t))'$ are vectors of dimension 3, and

$$\Pi_1 = \begin{bmatrix} 0.7 & -0.5 & 0.2 \\ 0.0 & -0.1 & 0.6 \\ -0.8 & -0.1 & 0.5 \end{bmatrix}, \qquad \mathrm{E}[a(t)a(t)'] = I_3 . \tag{13}$$

It is easy to see that all the roots of the equation $\det(I_3 - \Pi_1 z) = 0$ are outside the unit circle, so that the model is stationary. Furthermore, the innovations $a(t)$ have an identity covariance matrix so that there is no need to orthogonalize them to obtain a variance decomposition.

Since

$$X_2(t) = 0.1\, X_2(t-1) - 0.6\, X_3(t-1) + a_2(t) ,$$

it is clear that X_1 does not cause X_2 in the sense of Granger. On the other hand, when we look at the MA representation of $X_2(t)$,

$$X_2(t) = a_2(t) + \sum_{k=1}^{\infty} [\psi_{21k}a_1(t-k) + \psi_{22k}a_2(t-k) + \psi_{23k}\, a_3(t-k)] ,$$

Table 1
Impulse responses ψ_{21k} and variance proportions $p_{21}(k)$ from X_1 to X_2 in the trivariate AR(1) model (12)

k	1	2	3	4	5	10	25	50	100
ψ_{21k}	0	−0.480	−0.528	−0.365	−0.275	−0.166	−0.027	−0.001	0.000
$p_{21}(k)$	0	0.139	0.256	0.295	0.314	0.369	0.388	0.389	0.389

where $\psi_k = \Pi_1^k = [\psi_{ijk}]_{i,j=1,2,3}$, we see that the coefficients ψ_{21k} of the innovations $a_1(t-k)$, $k \geq 1$, are quite sizeable; see Table 1. Correspondingly, the proportion of the k-step-ahead forecast error variance of $X_2(t)$ due to the innovations $a_1(t)$, i.e.

$$p_{21}(k) = \sum_{h=0}^{k} \psi_{21h}^2 \Big/ \Big(\sum_{j=1}^{3} \sum_{h=0}^{k} \psi_{2jh}^2 \Big) ,$$

is different from zero; for example, for $k = 50$, it is equal to 0.389, clearly an important proportion of the total variance of $X_2(t)$.

Conversely, if we consider the trivariate MA(1) model,

$$X(t) = a(t) - \Pi_1 a(t-1), \quad t \in \mathbb{Z} ,$$

where Π_1 and $a(t)$ satisfy (13), the impulse responses of $a_1(t-k)$, $k \geq 0$, on $X_2(t)$ are all zero so that the proportion $p_{21}(k)$ of the k-step-ahead forecast error variance of $X_2(t)$ is zero, irrespective of the value of $k (k \geq 1)$. On the other hand, it is clear that X_1 causes X_2 in the sense of Granger.

5. Concluding remarks

In general, the characterization of non-causality from X_1 to X_2 given in Proposition 1 [condition (8)] leads one to consider non-linear restrictions on the MA coefficients of the model. It is easy to see that these restrictions involve multilinear forms in the MA coefficients (i.e. quadratic or higher order forms). In special cases, such as MA models of finite order (with possibly other restrictions on their coefficients), condition (8) may lead to relatively simple restrictions, which may be tested fairly easily with Wald-type or likelihood-ratio tests (under appropriate regularity conditions). It is also worthwhile noting that the sufficient conditions (10) or (11) and the necessary condition $\psi_{211} = 0$ are also linear in the MA coefficients, hence relatively easy to test. For AR models of finite order, however, such simplifications are unlikely to occur. Furthermore, it is well known that standard asymptotic theory may not work for tests of multilinear restrictions; for an illustration, see Boudjellaba et al. (1992).

In contrast with (8), the equivalent characterization $\Pi_{21}(z) = 0$ always involves *linear* restrictions (on the AR coefficients). Consequently, when the model is estimated in autoregressive form, which is typically the case for VAR models of finite order, the characterization $\Pi_{21}(z) = 0$ clearly provides a simpler and more natural parameterization for testing Granger non-causality. Even though the asymptotic distribution of estimated impulse responses derived from VAR models can be established under general regularity conditions [see, for example, Baillie (1987) and Lütkepohl (1990, 1991)], the impulse responses are non-linear transformations of the autoregressive coefficients Π_k, $k \geq 1$, and there is no advantage in considering such impulses to test Granger causality. It is also important to remember that the often-used characterization $\psi_{21}(z) = 0$ can be misleading when a third vector of variable is used to forecast: it is *neither necessary nor sufficient* to have Granger non-causality from X_1 to X_2. Furthermore, in this case, the proportion of the variance of X_2 accounted for by the innovations of X_1 is *not a measure of Granger causal priority* from X_1 to X_2. If it is related to 'causality', the characterization $\psi_{21}(z) = 0$ must involve a different notion of causality.

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